

- 1.) A line L passes through $A(1, -1, 2)$ and is parallel to the line $\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} + s \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

(a) Write down a vector equation for L in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

(2)

The line L passes through point P when $t = 2$.

(b) Find

(i) \overrightarrow{OP} ;

(ii) $|\overrightarrow{OP}|$.

(4)

(Total 6 marks)

- 2.) The following diagram shows the obtuse-angled triangle ABC such that

$$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix} \text{ and } \overrightarrow{AC} = \begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix}.$$

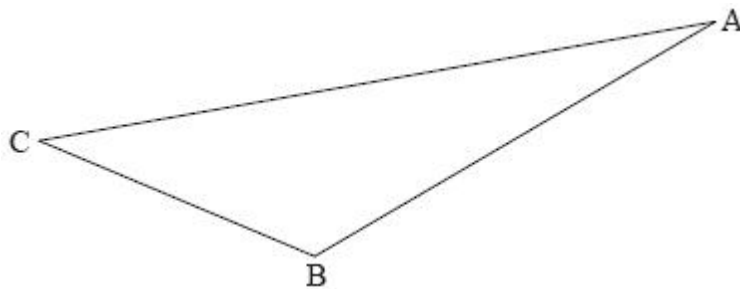


diagram not to scale

(a) (i) Write down \overrightarrow{BA} .

(ii) Find \overrightarrow{BC} .

(3)

(b) (i) Find $\cos \hat{ABC}$.

(ii) Hence, find $\sin \hat{ABC}$.

(7)

The point D is such that $\overrightarrow{CD} = \begin{pmatrix} -4 \\ 5 \\ p \end{pmatrix}$, where $p > 0$.

(c) (i) Given that $|\overrightarrow{CD}| = \sqrt{50}$, show that $p = 3$.

(ii) Hence, show that \overrightarrow{CD} is perpendicular to \overrightarrow{BC} .

(6)
(Total 16 marks)

- 3.) The following diagram shows quadrilateral ABCD, with $\overrightarrow{AD} = \overrightarrow{BC}$, $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

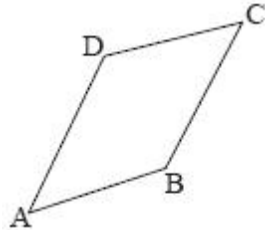


diagram not to scale

- (a) Find \overrightarrow{BC} . (2)
- (b) Show that $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$. (2)
- (c) Show that vectors \overrightarrow{BD} and \overrightarrow{CA} are perpendicular. (3)

(Total 7 marks)

- 4.) Line L_1 passes through points A(1, -1, 4) and B(2, -2, 5).

- (a) Find \overrightarrow{AB} . (2)
- (b) Find an equation for L_1 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. (2)

Line L_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

- (c) Find the angle between L_1 and L_2 . (7)
- (d) The lines L_1 and L_2 intersect at point C. Find the coordinates of C. (6)

(6)
(Total 17 marks)

5.) The line L_1 is represented by the vector equation $\mathbf{r} = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$.

A second line L_2 is parallel to L_1 and passes through the point B(-8, -5, 25).

- (a) Write down a vector equation for L_2 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. (2)

A third line L_3 is perpendicular to L_1 and is represented by $\mathbf{r} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$.

- (b) Show that $k = -2$. (5)

The lines L_1 and L_3 intersect at the point A.

- (c) Find the coordinates of A. (6)

The lines L_2 and L_3 intersect at point C where $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$.

- (d) (i) Find \overrightarrow{AB} .

- (ii) Hence, find $|\overrightarrow{AC}|$.

(5)
(Total 18 marks)

6.) Let $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

- (a) Find \overrightarrow{BC} . (2)

- (b) Find a unit vector in the direction of \overrightarrow{AB} . (3)

- (c) Show that \overrightarrow{AB} is perpendicular to \overrightarrow{AC} . (3)
(Total 8 marks)

7.) In this question, distance is in metres.

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, p seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$

(a) (i) Write down the coordinates of A.

(ii) Find the speed of the airplane in m s^{-1} .

(4)

(b) After seven seconds the airplane passes through a point B.

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds.

(5)

(c) Airplane 2 passes through a point C. Its position q seconds after it passes through C is

given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}, a \in \mathbb{R}.$

The angle between the flight paths of Airplane 1 and Airplane 2 is 40° . Find the two values of a .

(7)

(Total 16 marks)

8.) The diagram shows quadrilateral ABCD with vertices A(1, 0), B(1, 5), C(5, 2) and D(4, -1).

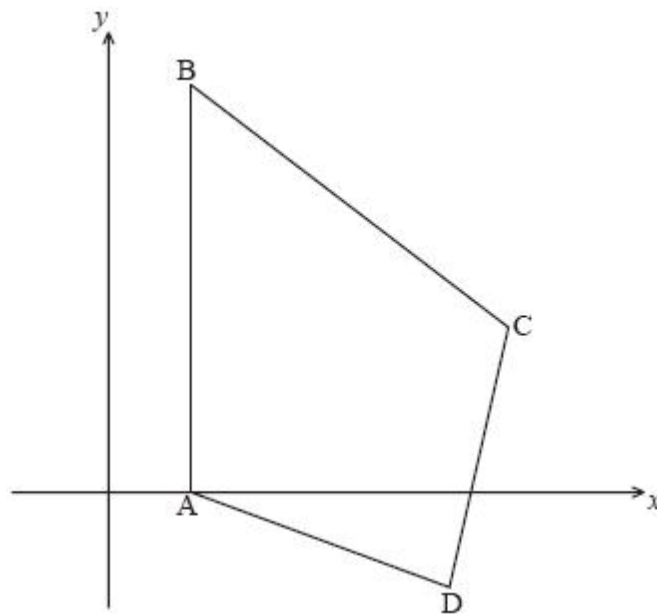


diagram not to scale

(a) (i) Show that $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

(ii) Find \overrightarrow{BD} .

(iii) Show that \overrightarrow{AC} is perpendicular to \overrightarrow{BD} .

(5)

The line (AC) has equation $\mathbf{r} = \mathbf{u} + s\mathbf{v}$.

(b) (i) Write down vector \mathbf{u} and vector \mathbf{v} .

(ii) Find a vector equation for the line (BD).

(4)

The lines (AC) and (BD) intersect at the point P(3, k).

(c) Show that $k = 1$.

(3)

(d) **Hence** find the area of triangle ACD.

(5)

(Total 17 marks)

9.) Let $\mathbf{v} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$, for $k > 0$. The angle between \mathbf{v} and \mathbf{w} is $\frac{\pi}{3}$.

Find the value of k .

(Total 7 marks)

10.) The vertices of the triangle PQR are defined by the position vectors

$$\overrightarrow{OP} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and } \overrightarrow{OR} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}.$$

(a) Find

(i) \overrightarrow{PQ} ;

(ii) \overrightarrow{PR} .

(3)

(b) Show that $\cos \hat{RPQ} = \frac{1}{2}$.

(7)

(c) (i) Find $\sin \hat{RPQ}$.

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$.

(6)

(Total 16 marks)

11.) Find the cosine of the angle between the two vectors $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and $4\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}$.

(Total 6 marks)

12.) The line L_1 is parallel to the z -axis. The point P has position vector $\begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix}$ and lies on L_1 .

(a) Write down the equation of L_1 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

(2)

The line L_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$. The point A has position vector $\begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix}$.

(b) Show that A lies on L_2 .

(4)

Let B be the point of intersection of lines L_1 and L_2 .

(c) (i) Show that $\overrightarrow{OB} = \begin{pmatrix} 9 \\ 1 \\ 14 \end{pmatrix}$.

(ii) Find \overrightarrow{AB} .

(7)

(d) The point C is at (2, 1, -4). Let D be the point such that ABCD is a parallelogram. Find \overrightarrow{OD} .

(3)

(Total 16 marks)

13.) Two lines with equations $\mathbf{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$ intersect at the point P. Find the coordinates of P.

(Total 6 marks)

14.) (a) Let $\mathbf{u} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}$. Given that \mathbf{u} is perpendicular to \mathbf{w} , find the value of p .

(3)

(b) Let $\mathbf{v} = \begin{pmatrix} 1 \\ q \\ 5 \end{pmatrix}$. Given that $|\mathbf{v}| = \sqrt{42}$, find the possible values of q .

(3)

(Total 6 marks)

15.) Consider the points P(2, -1, 5) and Q(3, -3, 8). Let L_1 be the line through P and Q.

(a) Show that $\overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$.

(1)

(b) The line L_1 may be represented by $\mathbf{r} = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + s \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$.

(i) What information does the vector $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$ give about L_1 ?

(ii) Write down another vector representation for L_1 using $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$.

(3)

The point T (-1, 5, p) lies on L_1 .

(c) Find the value of p .

(3)

The point T also lies on L_2 with equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ q \end{pmatrix}$.

(d) Show that $q = -3$.

(3)

(e) Let θ be the **obtuse** angle between L_1 and L_2 . Calculate the size of θ .

(7)

(Total 17 marks)

16.) Consider the points A (1, 5, 4), B (3, 1, 2) and D (3, k , 2), with (AD) perpendicular to (AB).

(a) Find

(i) \overrightarrow{AB} ;

(ii) \overrightarrow{AD} , giving your answer in terms of k .

(3)

(b) Show that $k = 7$.

(3)

The point C is such that $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$.

(c) Find the position vector of C.

(4)

(d) Find $\cos \hat{ABC}$.

(3)

(Total 13 marks)

17.) Let $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. The vector $\mathbf{v} + p\mathbf{w}$ is perpendicular to \mathbf{w} .

Find the value of p .

(Total 7 marks)

18.) The point O has coordinates (0, 0, 0), point A has coordinates (1, -2, 3) and point B has coordinates (-3, 4, 2).

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find \hat{BAO} .

(8)

(b) The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1 .

(2)

(c) The line L_2 passes through A and is parallel to \overrightarrow{OB} .

(i) Find a vector equation for L_2 , giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

(ii) Point C ($k, -k, 5$) is on L_2 . Find the coordinates of C.

(6)

(d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C.

Find the value of p at C.

(2)
(Total 18 marks)

19.) The line L_1 is represented by $\mathbf{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $\mathbf{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T.

(Total 6 marks)

20.) A particle is moving with a constant velocity along line L . Its initial position is A(6, -2, 10). After one second the particle has moved to B(9, -6, 15).

(a) (i) Find the velocity vector, \overrightarrow{AB} .

(ii) Find the speed of the particle.

(4)

(b) Write down an equation of the line L .

(2)

(Total 6 marks)

21.) The diagram shows a parallelogram ABCD.

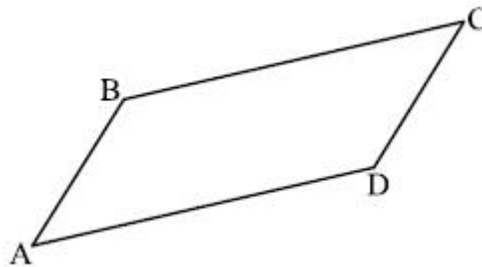


diagram not to scale

The coordinates of A, B and D are A(1, 2, 3), B(6, 4, 4) and D(2, 5, 5).

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$.

(ii) Find \overrightarrow{AD} .

(iii) Hence show that $\overrightarrow{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}$.

(5)

(b) Find the coordinates of point C.

(3)

(c) (i) Find $\overrightarrow{AB} \bullet \overrightarrow{AD}$.

(ii) Hence find angle A.

(7)

(d) Hence, or otherwise, find the area of the parallelogram.

(3)

(Total 18 marks)

22.) Consider the points A(5, 8), B(3, 5) and C(8, 6).

(a) Find

(i) \overrightarrow{AB} ;

(ii) \overrightarrow{AC} .

(3)

(b) (i) Find $\overrightarrow{AB} \bullet \overrightarrow{AC}$.

(ii) Find the sine of the angle between \overrightarrow{AB} and \overrightarrow{AC} .

(3)

(Total 6 marks)

23.) Two lines L_1 and L_2 are given by $\mathbf{r}_1 = \begin{pmatrix} 9 \\ 4 \\ -6 \end{pmatrix} + s \begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t \begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$.

(a) Let θ be the acute angle between L_1 and L_2 . Show that $\cos \theta = \frac{52}{140}$.

(5)

(b) (i) P is the point on L_1 when $s = 1$. Find the position vector of P.

(ii) Show that P is also on L_2 .

(8)

- (c) A third line L_3 has direction vector $\begin{pmatrix} 6 \\ x \\ -30 \end{pmatrix}$. If L_1 and L_3 are parallel, find the value of x .

(3)

(Total 16 marks)

- 24.) A triangle has its vertices at A(−1, 3), B(3, 6) and C(−4, 4).

- (a) Show that $\overrightarrow{AB} \bullet \overrightarrow{AC} = -9$.

(3)

- (b) Find \hat{BAC} .

(4)

(Total 7 marks)

- 25.) In this question, distance is in kilometres, time is in hours.

A balloon is moving at a constant height with a speed of 18 km h^{-1} , in the direction of the vector

$$\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}.$$

At time $t = 0$, the balloon is at point B with coordinates (0, 0, 5).

- (a) Show that the position vector \mathbf{b} of the balloon at time t is given by

$$\mathbf{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + t \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}.$$

(6)

At time $t = 0$, a helicopter goes to deliver a message to the balloon.

The position vector \mathbf{h} of the helicopter at time t is given by

$$\mathbf{h} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 49 \\ 32 \\ 0 \end{pmatrix} + t \begin{pmatrix} -48 \\ -24 \\ 6 \end{pmatrix}.$$

- (b) (i) Write down the coordinates of the starting position of the helicopter.

- (ii) Find the speed of the helicopter.

(4)

- (c) The helicopter reaches the balloon at point R.

- (i) Find the time the helicopter takes to reach the balloon.

- (ii) Find the coordinates of R.

(5)

(Total 15 marks)

26.) Consider the vectors $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} + \mathbf{j} - p\mathbf{k}$.

(a) Given that \mathbf{u} is perpendicular to \mathbf{v} find the value of p .

(b) Given that $q|\mathbf{u}|=14$, find the value of q .

(Total 6 marks)

27.) Points P and Q have position vectors $-5\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}$ and $-4\mathbf{i} + 9\mathbf{j} - 5\mathbf{k}$ respectively, and both lie on a line L_1 .

(a) (i) Find \overrightarrow{PQ} .

(ii) Hence show that the equation of L_1 can be written as

$$\mathbf{r} = (-5 + s)\mathbf{i} + (11 - 2s)\mathbf{j} + (-8 + 3s)\mathbf{k}. \quad (4)$$

The point R $(2, y_1, z_1)$ also lies on L_1 .

(b) Find the value of y_1 and of z_1 . (4)

The line L_2 has equation $\mathbf{r} = 2\mathbf{i} + 9\mathbf{j} + 13\mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

(c) The lines L_1 and L_2 intersect at a point T. Find the position vector of T. (7)

(d) Calculate the angle between the lines L_1 and L_2 . (7)
(Total 22 marks)

28.) In this question, distance is in metres, time is in minutes.

Two model airplanes are each flying in a straight line.

At 13:00 the first model airplane is at the point $(3, 2, 7)$. Its position vector after t minutes is

given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + t \begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}.$

(a) Find the speed of the model airplane. (2)

At 13:00 the second model airplane is at the point $(-5, 10, 23)$. After two minutes, it is at the point $(3, 16, 39)$.

(b) Show that its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$. (3)

(c) The airplanes meet at point Q.

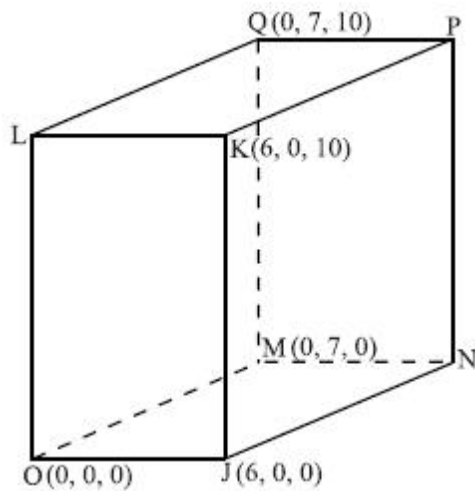
(i) At what time do the airplanes meet?

(ii) Find the position of Q. (6)

(d) Find the angle α between the paths of the two airplanes. (6)

(Total 17 marks)

29.) The diagram below shows a cuboid (rectangular solid) OJKLMNPQ. The vertex O is (0, 0, 0), J is (6, 0, 0), K is (6, 0, 10), M is (0, 7, 0) and Q is (0, 7, 10).



(a) (i) Show that $\overrightarrow{JQ} = \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$.

(ii) Find \overrightarrow{MK} . (2)

(b) An equation for the line (MK) is $\mathbf{r} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + s \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$.

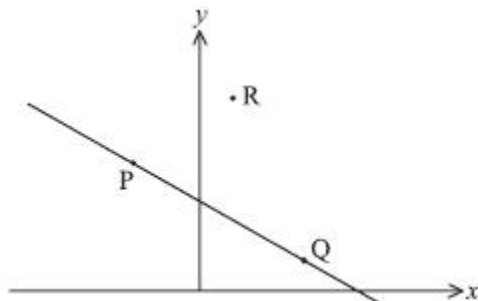
(i) Write down an equation for the line (JQ) in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

(ii) Find the acute angle between (JQ) and (MK). (9)

(c) The lines (JQ) and (MK) intersect at D. Find the position vector of D.

(5)
(Total 16 marks)

- 30.) The points $P(-2, 4)$, $Q(3, 1)$ and $R(1, 6)$ are shown in the diagram below.



- (a) Find the vector \overrightarrow{PQ} .
- (b) Find a vector equation for the line through R parallel to the line (PQ) .

(Total 6 marks)

- 31.) The position vector of point A is $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and the position vector of point B is $4\mathbf{i} - 5\mathbf{j} + 21\mathbf{k}$.

- (a) (i) Show that $\overrightarrow{AB} = 2\mathbf{i} - 8\mathbf{j} + 20\mathbf{k}$.
- (ii) Find the unit vector \mathbf{u} in the direction of \overrightarrow{AB} .
- (iii) Show that \mathbf{u} is perpendicular to \overrightarrow{OA} .

(6)

Let S be the midpoint of $[AB]$. The line L_1 passes through S and is parallel to \overrightarrow{OA} .

- (b) (i) Find the position vector of S .
- (ii) Write down the equation of L_1 .

(4)

The line L_2 has equation $\mathbf{r} = (5\mathbf{i} + 10\mathbf{j} + 10\mathbf{k}) + s(-2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$.

- (c) Explain why L_1 and L_2 are not parallel.

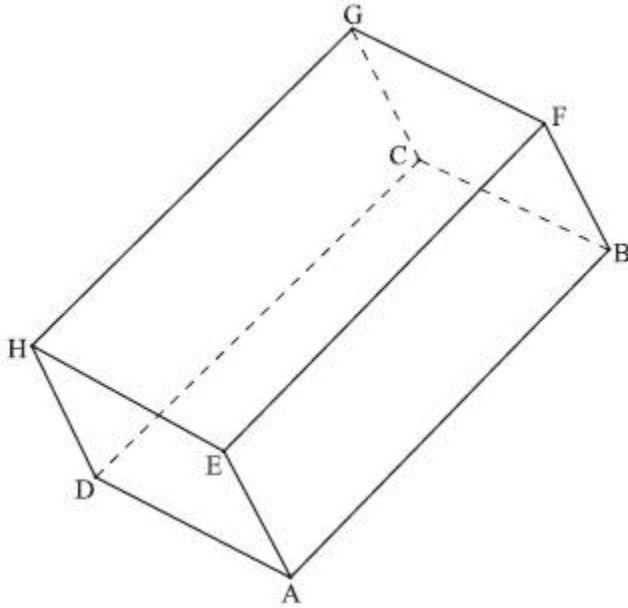
(2)

- (d) The lines L_1 and L_2 intersect at the point P . Find the position vector of P .

(7)
(Total 19 marks)

- 32.) The following diagram shows a solid figure $ABCDEFGH$. Each of the six faces is a

parallelogram.



The coordinates of A and B are A (7, -3, -5), B(17, 2, 5).

(a) Find

(i) \overrightarrow{AB} ;

(ii) $|\overrightarrow{AB}|$.

(4)

The following information is given.

$$\overrightarrow{AD} = \begin{pmatrix} -6 \\ 6 \\ 3 \end{pmatrix}, |\overrightarrow{AD}| = 9, \overrightarrow{AE} = \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix}, |\overrightarrow{AE}| = 6$$

(b) (i) Calculate $\overrightarrow{AD} \cdot \overrightarrow{AE}$.

(ii) Calculate $\overrightarrow{AB} \cdot \overrightarrow{AD}$.

(iii) Calculate $\overrightarrow{AB} \cdot \overrightarrow{AE}$.

(iv) Hence, write down the size of the angle between any two intersecting edges.

(5)

(c) Calculate the volume of the solid ABCDEFGH.

(2)

(d) The coordinates of G are (9, 14, 12). Find the coordinates of H.

(3)

(e) The lines (AG) and (HB) intersect at the point P.

Given that $\overrightarrow{AG} = \begin{pmatrix} 2 \\ 7 \\ 17 \end{pmatrix}$, find the acute angle at P.

(5)
(Total 19 marks)

33.) The line L passes through the points A (3, 2, 1) and B (1, 5, 3).

- Find the vector \overrightarrow{AB} .
- Write down a vector equation of the line L in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

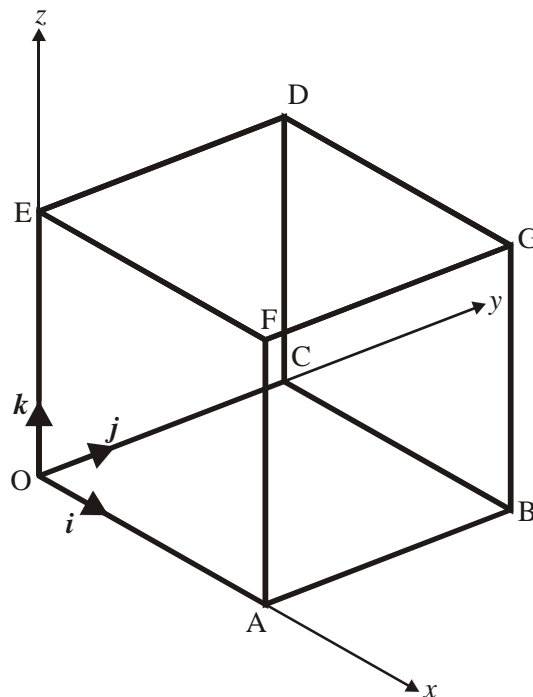
(Total 6 marks)

34.) The line L passes through A (0, 3) and B (1, 0). The origin is at O. The point R ($x, 3 - 3x$) is on L , and (OR) is perpendicular to L .

- Write down the vectors \overrightarrow{AB} and \overrightarrow{OR} .
- Use the scalar product to find the coordinates of R.

(Total 6 marks)

35.) The diagram shows a cube, OABCDEFG where the length of each edge is 5cm. Express the following vectors in terms of \mathbf{i}, \mathbf{j} and \mathbf{k} .



- \overrightarrow{OG} ;

(b) \overrightarrow{BD} ;

(c) \overrightarrow{EB} .

(Total 6 marks)

36.) A triangle has its vertices at A(−1, 3), B(3, 6) and C(−4, 4).

(a) Show that $\overrightarrow{AB} \bullet \overrightarrow{AC} = -9$

(b) Show that, to three significant figures, $\cos \hat{BAC} = -0.569$.

(Total 6 marks)

37.) In this question, distance is in kilometers, time is in hours.

A balloon is moving at a constant height with a speed of 18 km h^{-1} , in the

direction of the vector $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$

At time $t = 0$, the balloon is at point B with coordinates (0, 0, 5).

(a) Show that the position vector \mathbf{b} of the balloon at time t is given by

$$\mathbf{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + t \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}.$$

(6)

At time $t = 0$, a helicopter goes to deliver a message to the balloon. The position vector \mathbf{h} of the helicopter at time t is given by

$$\mathbf{h} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 49 \\ 32 \\ 0 \end{pmatrix} + t \begin{pmatrix} -48 \\ -24 \\ 6 \end{pmatrix}$$

(b) (i) Write down the coordinates of the starting position of the helicopter.

(ii) Find the speed of the helicopter.

(4)

(c) The helicopter reaches the balloon at point R.

(i) Find the time the helicopter takes to reach the balloon.

(ii) Find the coordinates of R.

(5)

(Total 15 marks)

- 38.) Find the cosine of the angle between the two vectors $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Working:

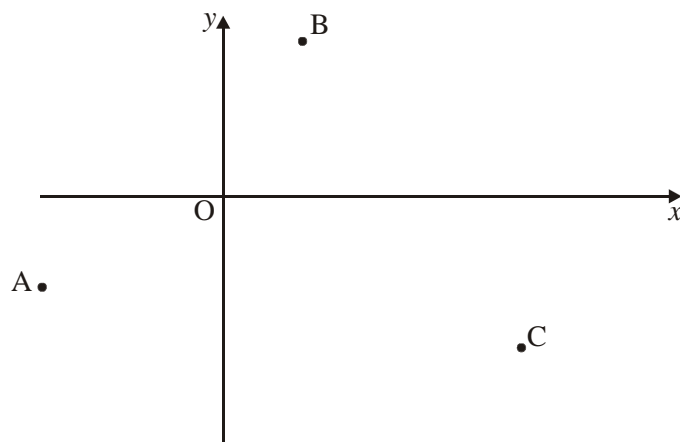
Answer:

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(Total 6 marks)

- 39.) In this question the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represents a displacement of 1 km east,
and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represents a displacement of 1 km north.

The diagram below shows the positions of towns A, B and C in relation to an airport O, which is at the point (0, 0). An aircraft flies over the three towns at a constant speed of 250 km h^{-1} .



Town A is 600 km west and 200 km south of the airport.
Town B is 200 km east and 400 km north of the airport.
Town C is 1200 km east and 350 km south of the airport.

(a) (i) Find \overrightarrow{AB} .

(ii) Show that the vector of length one unit in the direction of \overrightarrow{AB} is $\begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}$.

(4)

An aircraft flies over town A at 12:00, heading towards town B at 250 km h^{-1} .

Let $\begin{pmatrix} p \\ q \end{pmatrix}$ be the velocity vector of the aircraft. Let t be the number of hours in flight after 12:00.

The position of the aircraft can be given by the vector equation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -600 \\ -200 \end{pmatrix} + t \begin{pmatrix} p \\ q \end{pmatrix}.$$

(b) (i) Show that the velocity vector is $\begin{pmatrix} 200 \\ 150 \end{pmatrix}$.

(ii) Find the position of the aircraft at 13:00.

(iii) At what time is the aircraft flying over town B?

(6)

Over town B the aircraft changes direction so it now flies towards town C. It takes five hours to travel the 1250 km between B and C. Over town A the pilot noted that she had 17 000 litres of fuel left. The aircraft uses 1800 litres of fuel per hour when travelling at 250 km h^{-1} . When the fuel gets below 1000 litres a warning light comes on.

(c) How far from town C will the aircraft be when the warning light comes on?

(7)

(Total 17 marks)

40.) A boat B moves with constant velocity along a straight line. Its velocity vector is given by

$$\mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}.$$

At time $t = 0$ it is at the point $(-2, 1)$.

(a) Find the magnitude of \mathbf{v} .

(b) Find the coordinates of B when $t = 2$.

(c) Write down a vector equation representing the position of B, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

Working:

Answers:

(a)

(b)

(c)

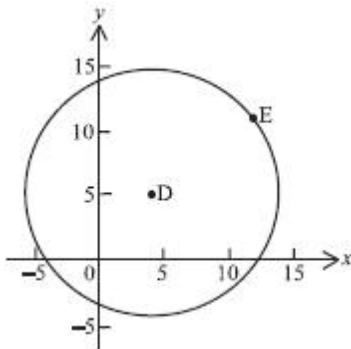
(Total 6 marks)

41.) Consider the point D with coordinates (4, 5), and the point E, with coordinates (12, 11).

(a) Find \overrightarrow{DE} . (2)

(b) Find $|\overrightarrow{DE}|$. (2)

(c) The point D is the centre of a circle and E is on the circumference as shown in the following diagram.



The point G is also on the circumference. \overrightarrow{DE} is perpendicular to \overrightarrow{DG} . Find the possible coordinates of G.

(8)
(Total 12 marks)

42.) Car 1 moves in a straight line, starting at point A (0, 12). Its position p seconds after it starts is given by $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 12 \end{pmatrix} + p \begin{pmatrix} 5 \\ -3 \end{pmatrix}$.

(a) Find the position vector of the car after 2 seconds. (2)

Car 2 moves in a straight line starting at point B (14, 0). Its position q seconds after it starts is given by $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 0 \end{pmatrix} + q \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

Cars 1 and 2 collide at point P.

- (b) (i) Find the value of p and the value of q when the collision occurs.
(ii) Find the coordinates of P.

(6)
(Total 8 marks)

43.) Two lines L_1 and L_2 have these vector equations.

$$L_1 : \mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + t(\mathbf{i} - 3\mathbf{j})$$

$$L_2 : \mathbf{r} = \mathbf{i} + 2\mathbf{j} + s(\mathbf{i} - \mathbf{j})$$

The angle between L_1 and L_2 is q . Find the cosine of the angle q .

Working:

Answer:

.....
(Total 6 marks)

44.) The points A and B have the position vectors $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ respectively.

- (a) (i) Find the vector \overrightarrow{AB} .
(ii) Find $|\overrightarrow{AB}|$.

(4)

The point D has position vector $\begin{pmatrix} d \\ 23 \end{pmatrix}$

- (b) Find the vector \overrightarrow{AD} in terms of d .

(2)

The angle $\hat{B}AD$ is 90° .

(c) (i) Show that $d = 7$.

(ii) Write down the position vector of the point D.

(3)

The quadrilateral ABCD is a rectangle.

(d) Find the position vector of the point C.

(4)

(e) Find the area of the rectangle ABCD.

(2)

(Total 15 marks)

45.) Points A, B, and C have position vectors $4\mathbf{i} + 2\mathbf{j}$, $\mathbf{i} - 3\mathbf{j}$ and $-5\mathbf{i} - 5\mathbf{j}$. Let D be a point on the x -axis such that ABCD forms a parallelogram.

(a) (i) Find \overrightarrow{BC} .

(ii) Find the position vector of D.

(4)

(b) Find the angle between \overrightarrow{BD} and \overrightarrow{AC} .

(6)

The line L_1 passes through A and is parallel to $\mathbf{i} + 4\mathbf{j}$. The line L_2 passes through B and is parallel to $2\mathbf{i} + 7\mathbf{j}$. A vector equation of L_1 is $\mathbf{r} = (4\mathbf{i} + 2\mathbf{j}) + s(\mathbf{i} + 4\mathbf{j})$.

(c) Write down a vector equation of L_2 in the form $\mathbf{r} = \mathbf{b} + t\mathbf{q}$.

(1)

(d) The lines L_1 and L_2 intersect at the point P. Find the position vector of P.

(4)

(Total 15 marks)

46.) The vector equations of two lines are given below.

$$\mathbf{r}_1 = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + l \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad \mathbf{r}_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

The lines intersect at the point P. Find the position vector of P.

Working:

Answer:

.....

(Total 6 marks)

47.) Consider the vectors $\mathbf{c} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{d} = 5\mathbf{i} - 12\mathbf{j}$.

Calculate the scalar product $\mathbf{c} \cdot \mathbf{d}$.

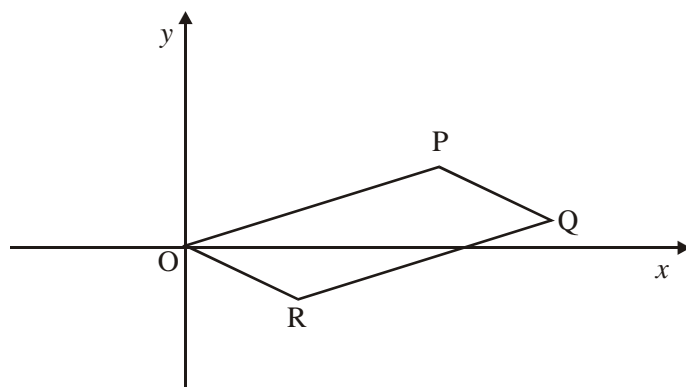
Working:

Answer:

.....

(Total 2 marks)

48.) The diagram shows a parallelogram OPQR in which $\overrightarrow{OP} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix} 10 \\ 1 \end{pmatrix}$.



(a) Find the vector \overrightarrow{OR} .

(3)

(b) Use the scalar product of two vectors to show that $\cos \hat{OPQ} = -\frac{15}{\sqrt{754}}$.

(4)

(c) (i) Explain why $\cos \hat{PQR} = -\cos \hat{OPQ}$.

(ii) Hence show that $\sin \hat{PQR} = \frac{23}{\sqrt{754}}$.

(iii) Calculate the area of the parallelogram OPQR, giving your answer as an integer.

(7)

(Total 14 marks)

49.) A vector equation for the line L is $\mathbf{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Which of the following are also vector equations for the same line L ?

A. $\mathbf{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

B. $\mathbf{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 6 \\ 2 \end{pmatrix}$.

C. $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

D. $\mathbf{r} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Working:

Answer:

.....
.....

(Total 6 marks)

- 50.) (a) Find the scalar product of the vectors $\begin{pmatrix} 60 \\ 25 \end{pmatrix}$ and $\begin{pmatrix} -30 \\ 40 \end{pmatrix}$.
- (b) Two markers are at the points P (60, 25) and Q (–30, 40). A surveyor stands at O (0, 0) and looks at marker P. Find the angle she turns through to look at marker Q.

Working:

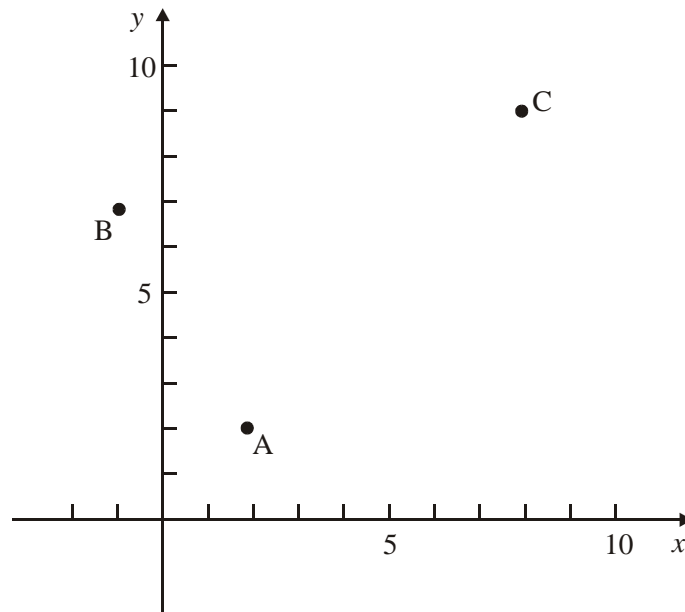
Answers:

- (a)
(b)

(Total 6 marks)

- 51.) The diagram shows points A, B and C which are three vertices of a parallelogram ABCD.

The point A has position vector $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$.



- (a) Write down the position vector of B and of C. (2)
- (b) The position vector of point D is $\begin{pmatrix} d \\ 4 \end{pmatrix}$. Find d . (3)
- (c) Find \overrightarrow{BD} . (1)

The line L passes through B and D.

- (d) (i) Write down a vector equation of L in the form

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t \begin{pmatrix} m \\ n \end{pmatrix}.$$

- (ii) Find the value of t at point B. (3)

- (e) Let P be the point (7, 5). By finding the value of t at P, show that P lies on the line L . (3)

- (f) Show that \overrightarrow{CP} is perpendicular to \overrightarrow{BD} . (4)
- (Total 16 marks)**

52.) A vector equation of a line is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} -2 \\ 3 \end{pmatrix}, t \in \mathbb{R}$.

Find the equation of this line in the form $ax + by = c$, where a , b , and $c \in \mathbb{Z}$.

Working:

Answer:

.....

(Total 6 marks)

53.) Three of the coordinates of the parallelogram STUV are S(−2, −2), T(7, 7), U(5, 15).

(a) Find the vector \overrightarrow{ST} and hence the coordinates of V. (5)

(b) Find a vector equation of the line (UV) in the form $\mathbf{r} = \mathbf{p} + l \mathbf{d}$ where $l \in \mathbb{R}$. (2)

(c) Show that the point E with position vector $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$ is on the line (UV), and find the value of l for this point. (2)

The point W has position vector $\begin{pmatrix} a \\ 17 \end{pmatrix}$, $a \in \mathbb{R}$.

(d) (i) If $|\overrightarrow{EW}| = 2\sqrt{13}$, show that one value of a is −3 and find the other possible value of a .

(ii) For $a = -3$, calculate the angle between \overrightarrow{EW} and \overrightarrow{ET} .

(10)
(Total 19 marks)

54.) Calculate the acute angle between the lines with equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Working:

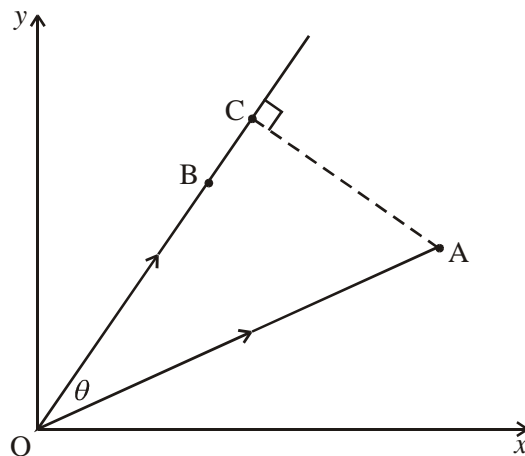
Answer:

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(Total 6 marks)

55.) The following diagram shows the point O with coordinates (0, 0), the point A with position vector $\mathbf{a} = 12\mathbf{i} + 5\mathbf{j}$, and the point B with position vector $\mathbf{b} = 6\mathbf{i} + 8\mathbf{j}$. The angle between (OA) and (OB) is q .

Diagram not to scale



Find

- (i) $|\mathbf{a}|$;
- (ii) a unit vector in the direction of \mathbf{b} ;
- (iii) the **exact** value of $\cos q$ in the form $\frac{p}{q}$, where, $p, q \in \mathbb{Z}$.

(Total 6 marks)

56.) Find the angle between the following vectors \mathbf{a} and \mathbf{b} , giving your answer to the nearest degree.

$$\mathbf{a} = -4\mathbf{i} - 2\mathbf{j}$$

$$\mathbf{b} = \mathbf{i} - 7\mathbf{j}$$

Working:

Answer:

.....

(Total 4 marks)

57.) In this question, a unit vector represents a displacement of 1 metre.

A miniature car moves in a straight line, starting at the point (2, 0).
After t seconds, its position, (x, y) , is given by the vector equation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0.7 \\ 1 \end{pmatrix}$$

- (a) How far from the point (0, 0) is the car after 2 seconds? (2)
- (b) Find the speed of the car. (2)
- (c) Obtain the equation of the car's path in the form $ax + by = c$. (2)

Another miniature vehicle, a motorcycle, starts at the point (0, 2), and travels in a straight line with constant speed. The equation of its path is

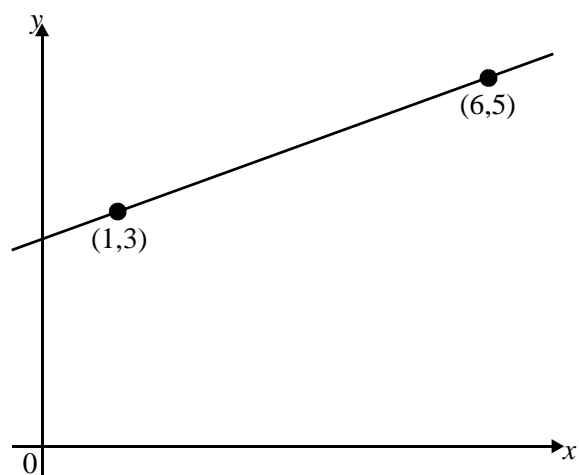
$$y = 0.6x + 2, \quad x \geq 0.$$

Eventually, the two miniature vehicles collide.

- (d) Find the coordinates of the collision point. (3)
- (e) If the motorcycle left point (0, 2) at the same moment the car left point (2, 0), find the speed of the motorcycle. (5)

(Total 14 marks)

58.) The diagram below shows a line passing through the points (1, 3) and (6, 5).



Find a vector equation for the line, giving your answer in the form

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + t \begin{pmatrix} c \\ d \end{pmatrix}, \text{ where } t \text{ is any real number.}$$

Working:

Answer:

.....

(Total 4 marks)

59.) The vectors $\begin{pmatrix} 2x \\ x-3 \end{pmatrix}$ and $\begin{pmatrix} x+1 \\ 5 \end{pmatrix}$ are perpendicular for two values of x .

- Write down the quadratic equation which the two values of x must satisfy.
- Find the two values of x .

Working:

Answers:

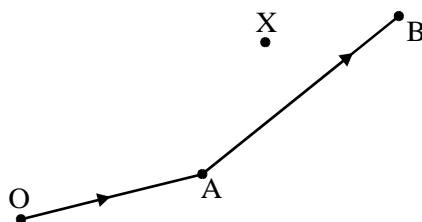
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- (a)
- (b)

(Total 4 marks)

- 60.) The diagram below shows the positions of towns O, A, B and X.

Diagram not to scale



Town A is 240 km East and 70 km North of O.
 Town B is 480 km East and 250 km North of O.
 Town X is 339 km East and 238 km North of O.

An airplane flies at a constant speed of 300 km h^{-1} from O towards A.

- (a) (i) Show that a unit vector in the direction of \overrightarrow{OA} is $\begin{pmatrix} 0.96 \\ 0.28 \end{pmatrix}$.

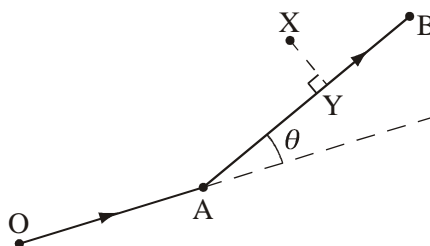
- (ii) Write down the velocity vector for the airplane in the form $\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$.

- (iii) How long does it take for the airplane to reach A?

(5)

At A the airplane changes direction so it now flies towards B. The angle between the original direction and the new direction is θ as shown in the following diagram. This diagram also shows the point Y, between A and B, where the airplane comes closest to X.

Diagram not to scale



- (b) Use the scalar product of two vectors to find the value of θ in degrees.

(4)

- (c) (i) Write down the vector \overrightarrow{AX} .

- (ii) Show that the vector $\mathbf{n} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ is perpendicular to \overrightarrow{AB} .

(iii) By finding the projection of \overrightarrow{AX} in the direction of \mathbf{n} , calculate the distance XY.

(6)

(d) How far is the airplane from A when it reaches Y ?

(3)

(Total 18 marks)

61.) Find the size of the angle between the two vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$. Give your answer to the nearest degree.

Working:

Answer:

.....

(Total 4 marks)

62.) A line passes through the point (4,−1) and its direction is perpendicular to the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Find the equation of the line in the form $ax + by = p$, where a , b and p are integers to be determined.

Working:

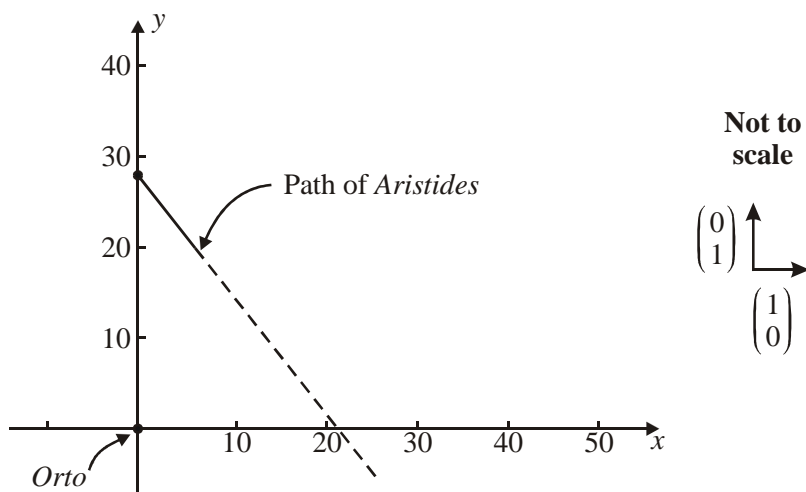
Answer:

.....

(Total 4 marks)

63.) In this question the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ km represents a displacement due east, and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ km represents a displacement due north.

The diagram shows the path of the oil-tanker *Aristides* relative to the port of *Orto*, which is situated at the point (0, 0).



The position of the *Aristides* is given by the vector equation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 28 \end{pmatrix} + t \begin{pmatrix} 6 \\ -8 \end{pmatrix}$$

at a time t hours after 12:00.

(a) Find the position of the *Aristides* at 13:00.

(2)

(b) Find

(i) the velocity vector;

(ii) the speed of the *Aristides*.

(4)

(c) Find a cartesian equation for the path of the *Aristides* in the form

$$ax + by = g.$$

(4)

Another ship, the cargo-vessel *Boadicea*, is stationary, with position vector $\begin{pmatrix} 18 \\ 4 \end{pmatrix}$ km.

(d) Show that the two ships will collide, and find the time of collision.

(4)

To avoid collision, the *Boadicea* starts to move at 13:00 with velocity vector $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$ km h⁻¹.

(e) Show that the position of the *Boadicea* for $t \geq 1$ is given by

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ -8 \end{pmatrix} + t \begin{pmatrix} 5 \\ 12 \end{pmatrix}$$

(2)

(f) Find how far apart the two ships are at 15:00.

(4)

(Total 20 marks)

64.) The vectors \mathbf{u} , \mathbf{v} are given by $\mathbf{u} = 3\mathbf{i} + 5\mathbf{j}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$.

Find scalars a , b such that $a(\mathbf{u} + \mathbf{v}) = 8\mathbf{i} + (b - 2)\mathbf{j}$.

Working:

Answer:

.....

(Total 4 marks)

65.) Find a vector equation of the line passing through $(-1, 4)$ and $(3, -1)$. Give your answer in the form $\mathbf{r} = \mathbf{p} + t\mathbf{d}$, where $t \in \mathbb{R}$.

Working:

Answer:

.....

(Total 4 marks)

66.) In this question, the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ km represents a displacement due east, and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ km a displacement due north.

Two crews of workers are laying an underground cable in a north–south direction across a

desert. At 06:00 each crew sets out from their base camp which is situated at the origin (0, 0). One crew is in a Toyundai vehicle and the other in a Chryssault vehicle.

The Toyundai has velocity vector $\begin{pmatrix} 18 \\ 24 \end{pmatrix} \text{ km h}^{-1}$, and the Chryssault has velocity vector $\begin{pmatrix} 36 \\ -16 \end{pmatrix} \text{ km h}^{-1}$.

- (a) Find the speed of each vehicle. (2)
- (b) (i) Find the position vectors of each vehicle at 06:30. (2)
- (ii) Hence, or otherwise, find the distance between the vehicles at 06:30. (3)
- (c) At this time (06:30) the Chryssault stops and its crew begin their day's work, laying cable in a northerly direction. The Toyundai continues travelling in the same direction at the same speed until it is exactly north of the Chryssault. The Toyundai crew then begin their day's work, laying cable in a southerly direction. At what time does the Toyundai crew begin laying cable? (4)
- (d) Each crew lays an average of 800 m of cable in an hour. If they work non-stop until their lunch break at 11:30, what is the distance between them at this time? (4)
- (e) How long would the Toyundai take to return to base camp from its lunch-time position, assuming it travelled in a straight line and with the same average speed as on the morning journey? (Give your answer to the nearest minute.) (5)
- (Total 20 marks)**

67.) The line L passes through the origin and is parallel to the vector $2\mathbf{i} + 3\mathbf{j}$. Write down a vector equation for L .

Working:

Answer:

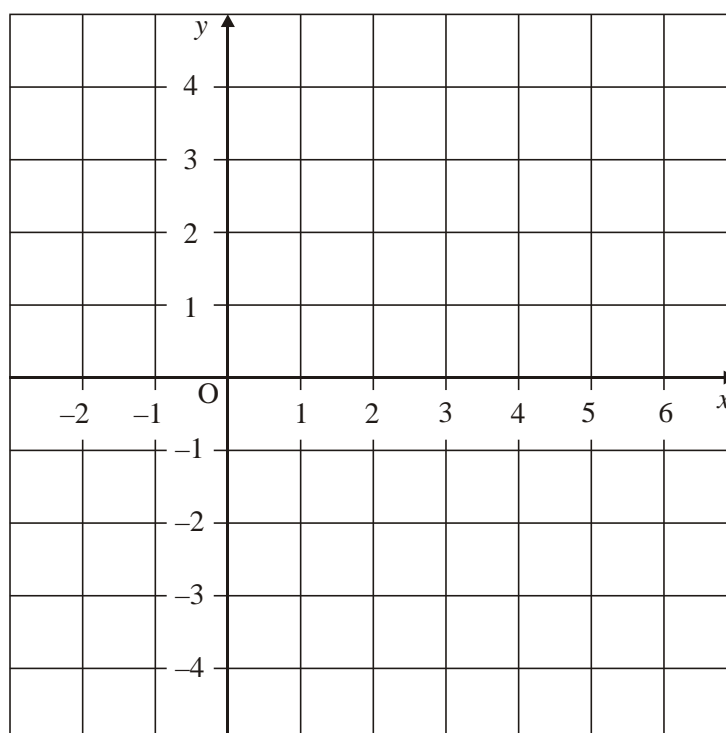
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(Total 4 marks)

68.) The triangle ABC is defined by the following information

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \quad \overrightarrow{AB} \cdot \overrightarrow{BC} = 0, \quad \overrightarrow{AC} \text{ is parallel to } \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

- (a) On the grid below, draw an accurate diagram of triangle ABC.



- (b) Write down the vector \overrightarrow{OC} .

Working:

Answer:

(b)

(Total 4 marks)

69.) In this question the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ km represents a displacement due east, and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ km represents a displacement due north.

The point (0, 0) is the position of *Shipple Airport*. The position vector r_1 of an aircraft *Air One* is given by

$$r_1 = \begin{pmatrix} 16 \\ 12 \end{pmatrix} + t \begin{pmatrix} 12 \\ -5 \end{pmatrix},$$

where t is the time in minutes **since** 12:00.

(a) Show that the *Air One* aircraft

(i) is 20 km from *Shipple Airport* at 12:00;

(ii) has a speed of 13 km/min.

(4)

(b) Show that a cartesian equation of the path of *Air One* is:

$$5x + 12y = 224.$$

(3)

The position vector r_2 of an aircraft *Air Two* is given by

$$r_2 = \begin{pmatrix} 23 \\ -5 \end{pmatrix} + t \begin{pmatrix} 2.5 \\ 6 \end{pmatrix},$$

where t is the time in minutes **since** 12:00.

(c) Find the angle between the paths of the two aircraft.

(4)

(d) (i) Find a cartesian equation for the path of *Air Two*.

(ii) Hence find the coordinates of the point where the two paths cross.

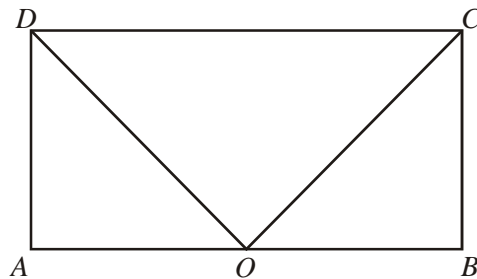
(5)

(e) Given that the two aircraft are flying at the same height, show that they do not collide.

(4)

(Total 20 marks)

70.) $ABCD$ is a rectangle and O is the midpoint of $[AB]$.



Express each of the following vectors in terms of \overrightarrow{OC} and \overrightarrow{OD}

(a) \overrightarrow{CD}

(b) \overrightarrow{OA}

(c) \overrightarrow{AD}

Working:

Answers:

(a)

(b)

(c)

(Total 4 marks)

71.) The vectors i , j are unit vectors along the x -axis and y -axis respectively.
The vectors $u = -i + 2j$ and $v = 3i + 5j$ are given.

(a) Find $u + 2v$ in terms of i and j .

A vector w has the same direction as $u + 2v$, and has a magnitude of 26.

(b) Find w in terms of i and j .

Working:

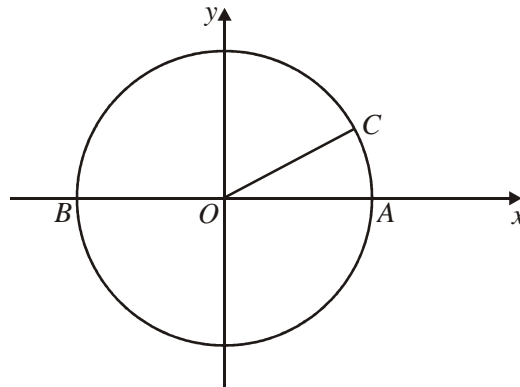
Answers:

(a)

(b)

(Total 4 marks)

- 72.) The circle shown has centre O and radius 6. \overrightarrow{OA} is the vector $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$, \overrightarrow{OB} is the vector $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$ and \overrightarrow{OC} is the vector $\begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix}$.



- Verify that A , B and C lie on the circle. (3)
 - Find the vector \overrightarrow{AC} . (2)
 - Using an appropriate scalar product, or otherwise, find the cosine of angle \hat{OAC} . (3)
 - Find the area of triangle ABC , giving your answer in the form $a\sqrt{11}$, where $a \in \mathbb{N}$. (4)
- (Total 12 marks)

- 73.) The quadrilateral $OABC$ has vertices with coordinates $O(0, 0)$, $A(5, 1)$, $B(10, 5)$ and $C(2, 7)$.

- Find the vectors \overrightarrow{OB} and \overrightarrow{AC} .
- Find the angle between the diagonals of the quadrilateral $OABC$.

Working:

Answers:

-
-

(Total 4 marks)

